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Introductory remarks

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THE ROYAL SOCIETY

Introductory remarks

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A brief review is given of the different physical aspects of ice and snow accretion on structures. Included is a general discussion of theoretical icing models currently employed to describe these phenomena.

Keywords: atmospheric icing; wet-snow accretion; icing models

1. General background

The science of ice accretion, in the widest sense in which the term is used today, is concerned with the physical mechanisms governing the adherence of atmospheric icing particles to a surface. Typical particles are the supercooled droplets of freezing fog, drizzle and rain; also snowflakes occurring in a wet-snow storm. Research in atmospheric icing is based on studies of hailstone growth and aircraft icing. Indeed, the commentary on the heat economy of hailstone growth as given by Ludlam (1951) contains the basic physics of the atmospheric ice-accretion process, as follows.

> When a hailstone falls through a cloud of supercooled droplets a substantial proportion of those in its path are caught and commence to freeze on its surface. The freezing is accompanied by the release of the latent heat of fusion and the surface temperature rises above that of the environment. The rate of freezing is controlled by the rate at which heat is transferred to the environment. The greatest rate of heat transfer occurs when the surface temperature reaches the fusion temperature, and corresponds to a maximum rate of freezing… it frequently happens that supercooled water is caught at such a rate that it cannot all be frozen, so that the liquid accumulates on the hailstone until drops are shed into its wake. When the surface of the hailstone is wet, ice is deposited in hard glassy form, but when impinging droplets freeze rapidly on contact the ice is opaque and fragile containing air enclosures.

The above modes of accretion are now referred to as glaze and rime, respectively.

In a wartime project, Taylor (1940) proposed the design of an icing wind tunnel to investigate the icing and de-icing of aircraft wings. Taylor was interested in determining conditions for the trajectories of droplets to retain similarity when the diameter, speed or density is changed. Taylor assumed the Stokes formula for the viscous drag on a droplet and, for further simplicity, considered the deflection of droplet trajectories in attached potential flow past a circular cylinder. He established an important result in ice-accretion kinetics, namely that there was a minimum droplet size below which the droplet could not impact on the cylinder surface. Adopting more realistic empirical droplet drag formulae for low wind speeds, Langmuir & Blodgett (1946) integrated the vector equation of motion for a droplet using numerical methods. From these results a theoretical database was constructed for droplet impaction velocities on the windward side of a cylinder. So, for the first time, theoretical predictions could be made for the initial icing intensities of rime accretion on a bare circular cylinder. These seminal investigations of Taylor (1940) and Langmuir & Blodgett (1946) soon became central to the construction of icing models used to simulate atmospheric icing on structures (see Imai 1953; Ackley & Templeton 1979; Lozowski *et al*. 1983; Makkonen 1981, 1984); regarding aircraft icing, important advances were also made by Cansdale & McNaughton (1977). These developments together with recent studies on wet-snow accretion will be examined in greater detail in § 2.

Perhaps at this point it is informative to provide some reasons for justifying the study of ice and snow accretion on structures. In cold regions, during the winter period, severe ice and snow storms are known to cause irreparable damage to the environment, telecommunication and electricity supply installations, and, moreover, make land, sea and air travel hazardous. Indeed, the social and economic impact of such storms may be considerable, as seen in the following examples. The exceptional freezing-rain event in Tennessee, USA, in 1951 (see Harlin 1952; Stallabrass 1982) caused 25 deaths, 500 non-fatal accidents, and the estimated cost of damage to the environment, etc., given in units of 1 million US dollars, was reported as follows: forests 56, communication and power lines 10, roads 15, fruit trees 4, buildings 4, livestock 33 and business 2 (units), respectively. More recently, in the 1998 Great Ice Storm in eastern Canada the damage to power systems alone was estimated at 2 billion Canadian dollars. Often, seemingly less extreme events than those mentioned above may give rise to serious problems in our local environment. Such an event, involving snow blizzard conditions during 8–9 December 1990, affected the grid system of the Midlands Electricity Regions of the UK (see the reports of Morgan (1991) and Bartlett & Holley (1991)). Wind speeds up to 50 mph were recorded and cylindrical sleeves on conductors and earth wires ranged from 75 to 200 mm in diameter. Conductors and earth wires sagged at near ground level due to wet-snow and wind loads. In a period of 14 h, the damage extended over some 3000 square miles of the Midlands and 27 million hours of electricity supply were lost to customers. Coupled with these were delays in road, rail and air transport and many other difficulties in domestic and work situations.

In what follows, a general review will be given of the physical processes involved in ice and wet-snow accretion, and this leads to a discussion of various icing models with some applications.

2. Physical nature of the ice- and snow-accretion processes

(*a*) *Ice accretion*

The ice-accretion process at the surface is assumed to be in thermodynamic equilibrium. Thermal exchanges between the impinging droplets, the surface and the environment are taken to be instantaneous and the local surface temperature is the equilibrium temperature at that location. An analysis of the heat and mass transfer mechanisms controlling the equilibrium state of iced aircraft surfaces was given by Messinger (1953), and that for a circular cylinder by Ludlam (1951).

The heat-balance equation is simply the total sum of the local heat fluxes at the surface, equated to zero. These are as follows:

(i) the latent heat released on freezing some or all of the impinging water,

(ii) the aerodynamic heating of the air,

(iii) the kinetic energy of impinging water,

(iv) the heat released on surface cooling from fusion to surface temperature,

(v) the sensible heat flux between the accretion and airstream (conduction and convection),

(vi) evaporative heat flux from the accretion surface,

(vii) heat loss in warming impinging supercooled water to the fusion temperature,

(viii) energy gained due to short-wave radiation,

(ix) energy loss from long-wave radiation,

(x) heat loss by conduction between the ice and substrate, and, finally,

(xi) heat flux terms related to the downstream flow and heat transfer of runback water.

The evaluation of the above heat fluxes for accretion on a circular cylinder and on aerofoils is described in Lozowski *et al.* (1983) and Cansdale & Gent (1983), respectively (see also Makkonen 1984; Brown & Krishnasamy 1984).

Dry growth or rime ice accretion usually takes place at lower liquid water content and lower air temperature. The supercooled droplet on adhering to the surface absorbs heat from it and warms up to the fusion temperature, releases its latent heat into the environment, and then cools down to the surface temperature. During this mode of accretion, the heat-balance equation predicts the surface temperature of the ice surface.

At higher liquid water content, and at air temperatures approaching the fusion temperature, conditions may exist that promote wet growth. Providing the energy loss from the accreting surface is less than the latent heat of fusion released by freezing droplets, the ice surface will remain at the fusion temperature and is covered by a film of water. For this mode of accretion, the heat-balance equation predicts the freezing fraction of the impinging water. In the case of an accreting circular cylinder, the remaining water runs back towards the (viscous) airflow separation line on the ice surface, where it is shed or freezes; on the other hand, when the airflow during accretion is approximated by attached potential flow, the runback water can extend to the shoulder of the cylinder. In Lozowski *et al.* (1983), the introduction of the concept of runback water, and the study of its interaction with the heat- and mass-transfer processes during wet growth, was germaine to the development of recent icing models. In Szilder *et al.* (1987), a fully time-dependent model for cylinder icing was constructed that employed finite-element methods to track the evolution of a dry- or wet-accretion surface. Two-dimensional and time-dependent models for rime and glaze accretion on a circular cylinder, which reformulate the process as a moving-boundary problem with change of phase, have also been developed by Poots & Skelton (1992*a*, *b*). However, it is clear that the simulation of the accretion process using mathematical/computational models may prove difficult for certain environmental conditions, such as those which promote the evolution of rime horns at the shoulders of the cylinder and the simultaneous emergence of glaze, for example.

(b) Wet-snow accretion

Wet-snow precipitation occurs at temperatures just above the fusion temperature. The snow particle is an agglomeration of snowflakes and is a mixture of ice, water and air. In controlled wind-tunnel experiments using real snowflakes, Wakahama *et al*. (1977) employed photographic techniques to show that snow-particle trajectories about a circular cylinder prior to impaction are rectilinear. They also observed that on impact the snow particles suffer fragmentation, with some of the snow ricocheting into the airflow. The fragmentation and adhesion of snowflakes was observed to be a random process; moreover, snow particles from the surface were often dislodged during the impaction process. Interpretation of their results gave rise to the concept of a wet-snow accretion factor, which was defined as the ratio of the mass flow of snow particles that stick or adhere to the windward side of the cylinder to the mass flow that would be experienced by the cylinder prior to impaction. The accretion factor is usually found to be small, and in the wind-tunnel experiments typical values were of $O(1/5)$. Wakahama *et al*. (1977) also demonstrated the major role played by the wind in producing large cylindrical sleeve accretions of wet snow. In particular, the wind forces hold the accretion to the cylinder, allowing for rapid growth and its densification; also, snow torque due to the eccentric snow loading on the windward side caused the accretion to rotate around the cylinder, promoting further accretion.

Wakahama *et al*. (1977) related snow-accretion adhesion to capillary forces within the snow matrix, while Kemp (1980) and Ryder (1981) suggested that it was due to particle freezing caused by evaporative cooling. However, Colbeck & Ackley (1982) showed that it was inter-particle ice bonding that was responsible for the adhesive strength of wet snow. They argued that the effect of the wind was to exert viscous shear and normal stresses on the surface of the accumulation and that such forces would be much greater than those expected from internal capillary forces. They further postulated that during the process of metamorphism occurring on the cylinder, the ice grains begin to round and large particles would grow at the expense of smaller particles. In this connection it is of interest to note (see Admirat & Sakamoto 1988) that when immersed in a snow layer at fusion temperature ice crystals undergo metamorphism in some 10 h, while falling in the atmosphere, at slightly positive air temperature, only a few minutes are needed.

In wet-snow accretion on transmission-line conductors the liquid water content of the snow matrix obviously controls the strength of internal capillary forces and promotes contact between ice granules, leading to ice bonding. During the process of metamorphism, the liquid water content of the snow matrix increases with time and, on reaching a level of 20–40%, the internal forces are greatly weakened, causing snow shedding by gravitational and aerodynamic forces. Consequently, critical meteorological conditions (precipitation rate, wind velocity and air temperature) exist for which sawtooth transient snow loading of the transmission line may be observed (see Admirat & Sakamoto 1988).

The first thermodynamical model for the prediction of the liquid water content in a wet-snow sleeve was due to Grenier *et al*. (1985) and this work has given further insight into the formation of wet-snow overloads and the process of snow shedding. Assuming a state of thermodynamic equilibrium on the accretion surface, the heat balance for the accumulated snow is the sum of the relevant thermal exchanges equated to zero, namely the latent heat required to melt the snow, the evaporative/condensation heat flux for the accretion surface, and the convective heat flux from the airstream. To these may be added a Joule heat flux if its use either to avoid or limit accretion is under investigation. Other heat fluxes, such as those representing radiation, aerodynamic heating and kinetic energy of snowflakes, are usually neglected. Thus, the heat-balance equation models the rate of melting of the snow matrix and, hence, determines its liquid water content as a function of time.

3. Aspects of ice- and snow-accretion models

As previously discussed, the thermodynamic state of the accretion governs the accretion process. However, there are many other processes to be included in the construction of a mathematical/computational icing model. The evolution of the icing surface affects the airflow in the vicinity of the accreting object, thus altering trajectory paths of icing particles and their impaction velocities and locations. Moreover, it alters the aerodynamic flow characteristics of the accreting object and, in some instances, may instigate aerodynamic instabilities. The calculation of the airflow is based on the assumption that it is quasi-steady during a time-step of the icing process. In the case of an accreting circular cylinder, the airflow may be approximated by attached potential flow and obtained using boundary-element methods; more generally, at each icing time-step, a computer fluid dynamics turbulent flow code may be used.

For rime- and glaze-accretion models it is necessary to input the basic meteorological parameters: the wind velocity, temperatures of the airflow and supercooled droplets, liquid water content of the air together with droplet size and distribution. Empirical formulae for the density of ice as a function of the droplet size, droplet speed of impact and the ice surface temperature are available from experiments by Macklin (1962). Finally, the evolution of the ice surface is computed at icing sites distributed over the icing surface and procedures for the tracking of runback water and its interaction with downstream icing are given in the original paper of Lozowski *et al*. (1983). Regarding ice accretion on an overhead line conductor of finite length and finite torsional stiffness, the eccentric iceload on the windward side causes the conductor to rotate. Consequently, axial growth advancing out into the wind occurs near the (fixed) anchor on the tower, together with the progressive development of cylindrical sleeve growth towards the centre of the span. Following Admirat & Lapeyre (1988), the conductor rotation due to ice torque is determined on solving the nonlinear torsion equation. Conductor icing is clearly a good example of the interactions taking place during icing between the accreting surface of the structure, its evolving mechanical state and changing aerodynamic characteristics. Such dynamical rime-accretion models for a transmission-line conductor, which are time dependent and three dimensional, are described in Poots (1996).

In developing wet-snow accretion models, the following meteorological parameters are necessary as input: wind velocity, precipitation rate, airflow temperature, and the liquid water content of the impacting snowflakes. To this end, Admirat *et al*. (1988) constructed a useful database for transmission lines from field observations of wet-snow events in France and Japan. They calibrated a simple cylindrical-sleeve model on assigning values to the accretion factor and the accreted snow density, so as to obtain coincidence between the model predictions and field observations. This enabled correlations for the accretion factor and the snow density to be given simply as functions of the wind speed; an empirical formula was also recommended for the liquid-water content of incident snowflakes as a function of the air temperature. One further feature is encountered in the modelling of wet-snow accretion. If it is assumed that the accretion factor of the snow surface is constant, then, for example, in the case of accretion on a fixed circular cylinder, the theoretical model predicts a semicircular snowfront growing into the wind. This is because the trajectories of snowflakes are rectilinear. Indeed the assumption of a constant accretion factor will always result in the replication of the original shape of the structure. However, field observations and snow wind-tunnel experiments show that the axial growth on a cylinder is roughly semi-elliptical in shape. Obviously, agreement between observations and theory could be achieved if the accretion factor was allowed to vary on the surface. Bearing in mind the accretion kinetics of snowflake capture and the geometry of the accreting surface, it was shown in Poots (1996) that if the accretion factor is assumed to be proportional to the cosine of the angle between an incident trajectory and the normal to the surface at an impaction site, then the theoretical and observed wet-snow accretion surfaces on a cylinder are in exact agreement; furthermore, in the above theoretical investigation it was possible to obtain an exact analytical solution of the nonlinear evolution equation for axial growth. Based on the above assumptions, three-dimensional and time-dependent dynamical models of wet-snow accretion have been developed in Poots (1996).

4. Examples of ice and wet-snow accretion on structures

A general statement of the research area is given in List (1977). The roles of various atmospheric parameters in determining ice-accretion intensity on structures near the ground are discussed in Makkonen (1981). Here, a brief introduction to some of the main applications is presented.

(*a*) *Marine icing*

Freezing spray is the main cause of superstructure icing on ships. Spray icing results in high icing rates and iceloads that may be large enough to alter the location of the centre of gravity of a ship and, in some incidents, to cause a ship to capsize. Makkonen (1987) proposed a method for modelling salinity effects, and in Zakrewski *et al.* (1988) a ship icing model was developed to calculate spray impact temperature, pure-ice instantaneous growth rate, and the time-averaged ice growth rate for saline spongy accretions. It was also shown that the icing rate at an icing site depends on its location and orientation, since it receives brine and heat transport coming from more elevated icing sites of the superstructure.

(*b*) *Aircraft icing*

An excellent review of helicopter rotor ice-accretion and protection research is available in Cansdale (1981). Mathematical/computational models of electrothermal de-icing of rotor blades are proposed in Cansdale & Gent (1983). An overview of icing research on aerofoils appears in Gufford *et al.* (1988) and deals with theoretical model predictions and their validity in comparison with wind-tunnel experiments.

(*c*) *Icing of overhead transmission lines*

Considerable advances have now been made in the construction of theoretical models for ice and wet-snow accretion on overhead-line conductors. They have been developed in the light of field observations undertaken by several electrical utilities operating in cold regions. Ice-accretion research is reviewed in Lozowski & Gayet (1988) and research into wet-snow accretion is reviewed in Admirat & Sakamoto (1988) (see also Poots 1996).

5. Concluding remarks

The papers that follow this one reflect the rapid progress taking place in icing research and its social and economic benefits to cold regions.

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